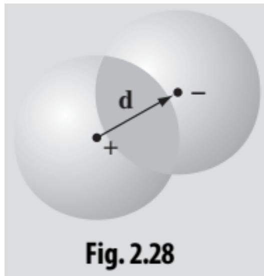
Problem 2.19

Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (Fig. 2.28). Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value. [*Hint:* Use the answer to Prob. 2.13.]

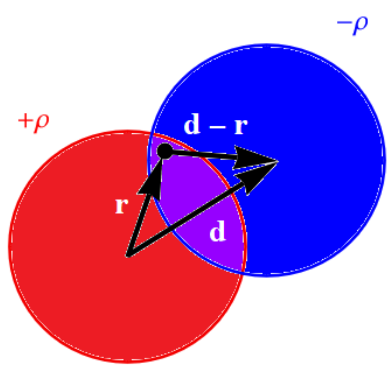


Solution

According to Problem 2.13, the electric field around a solid ball with a uniform volume charge density ρ is

$$\mathbf{E} = \begin{cases} \dfrac{\rho}{3\epsilon_0} r\hat{\mathbf{r}} & \text{if } r < R \\[2ex] \dfrac{\rho}{3\epsilon_0}\dfrac{R^3}{r^2}\hat{\mathbf{r}} & \text{if } r > R \end{cases} = \begin{cases} \dfrac{\rho}{3\epsilon_0} r\left(\dfrac{\mathbf{r}}{r}\right) & \text{if } r < R \\[2ex] \dfrac{\rho}{3\epsilon_0}\dfrac{R^3}{r^2}\left(\dfrac{\mathbf{r}}{r}\right) & \text{if } r > R \end{cases} = \begin{cases} \dfrac{\rho}{3\epsilon_0}\mathbf{r} & \text{if } r < R \\[2ex] \dfrac{\rho}{3\epsilon_0}\dfrac{R^3}{r^3}\mathbf{r} & \text{if } r > R \end{cases},$$

where $\mathbf{r}$ is the position vector from the ball's center to the point where we want to know the electric field. In the region of overlap the $r < R$ condition applies for both the positive ball and the negative ball.



The electric field at $\mathbf{r}$ due to the positive and negative balls alone are respectively

$$\mathbf{E}_+ = \frac{+\rho}{3\epsilon_0}\mathbf{r} \quad \text{and} \quad \mathbf{E}_- = \frac{-\rho}{3\epsilon_0}(\mathbf{r} - \mathbf{d}).$$

$\mathbf{r} - \mathbf{d}$ has the same magnitude as $\mathbf{d} - \mathbf{r}$ but points toward the point of interest.

By the principle of superposition, the total electric field at $\mathbf{r}$ is the vector sum of these fields.

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_+ + \mathbf{E}_- \\ &= \frac{+\rho}{3\epsilon_0}\mathbf{r} + \frac{-\rho}{3\epsilon_0}(\mathbf{r} - \mathbf{d}) \\ &= \cancel{\frac{\rho}{3\epsilon_0}\mathbf{r}} - \cancel{\frac{\rho}{3\epsilon_0}\mathbf{r}} + \frac{\rho}{3\epsilon_0}\mathbf{d}\end{aligned}$$

Therefore, in the region of overlap,

$$\mathbf{E} = \frac{\rho}{3\epsilon_0}\mathbf{d},$$

which is constant.